

Gambling strategies and gamblers' expectations in the roulette game: a real data validation

Le aspettative degli scommettitori al gioco della roulette: alcune verifiche empiriche

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Riassunto: *Il gioco della roulette ha stimolato la creazione di sistemi di puntate più di ogni altro gioco da casinò. L'obiettivo dichiarato da molti di questi sistemi è quello di ribaltare la condizione sfavorevole del giocatore e garantire una rendita media positiva. Sebbene sia ben noto in letteratura che questo risultato non può essere ottenuto, la decisione di come e quanto puntare permette di massimizzare la probabilità di raggiungere certi obiettivi prefissati dal giocatore. In questo lavoro si analizzano e si confrontano alcuni dei sistemi più noti e utilizzati nei casinò. La disponibilità delle uscite dei numeri di una singola roulette nel corso di un anno solare di una nota casa da gioco nazionale, permette la verifica della non distorsione della ruota e la perfetta aderenza delle uscite empiriche a quelle attese rispetto al corrispondente modello teorico uniforme discreto.*

Keywords: bet, gambling systems, Garçia system, permanencies, roulette game.

1. Introduction

The spread and popularity that games of chance have been having since ancient times, made their study and stochastic modelling evergreen topics, particularly after that, during the XVII century, they stimulated the first systematic works in the fields of theoretical probability. During the second half of the last century, a large number of works brought to the definition of general methods aimed to maximize the chances of beating the bank. Classic works concerning these topics are the books by Dubins e Savane (1976), Epstein (1967), and Levinson (1963). Even if it had been demonstrated (see, for example, the elegant results by Doob, 1953) that a gambler can not construct a series of bets which transform a game unfavourable for him into a favourable one (i.e. it is not possible to construct a super-fair bet from a combination of subfair ones), this consciousness is seldom possessed by “ordinary” players, nor that excludes the possibility of searching for an optimal game strategy. The usual situation in which the bank has bigger chances of victory than the player, determines, for the former, at least two apparent disadvantages for the bank: the player decides how to bet and also, keeping within certain limits, how much to bet.

In this paper some of the most diffused gambling strategies are analyzed under a stochastic perspective comparing their returns. Although the works concerning these topics are very numerous (see, among the others, Costantini and Monari, 2002 and Maitra and Sudderth,

1996) few of them can directly apply the theoretical models therein described on real data. In this work the empirical validation is carried out not on simulated data but on real come ups, demonstrating that the data supplied by a primary Italian gambling house (which asked to remain anonymous) are unbiased and that any subsequent study can legitimately make use of computer generated random come ups.

2. Verification of the unbiasedness of the roulette wheel

Empirical data consist of 87090 drawings of $k = 37$ possible come ups collected by the casino during a whole solar year in a daily distinction for one single wheel. The unbiasedness can be interpreted in two different ways: equiprobability of the k single numbers and absence of dependence among each come up and all the preceding ones. The first characteristic is verified through a distributional study weighing if the empirical distribution of numbers is comparable to a discrete uniform. This condition is not, by itself, sufficient to guarantee the randomness of come ups, because their order of apparition is equally relevant.

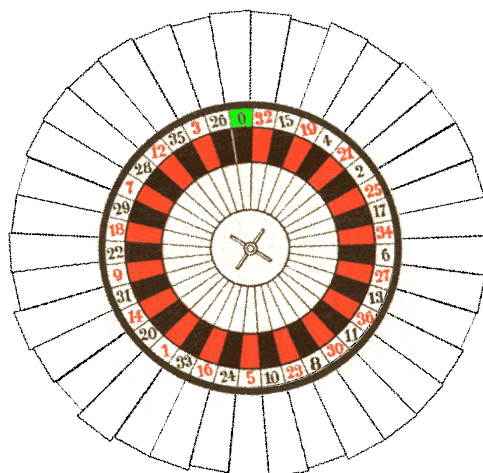
The most reasonable principle upon which the unbiasedness of the roulette wheel can be tested is the use of a goodness-of-fit test verifying the null hypothesis of discrete uniform distribution having support $D = \{0, 1, 2, \dots, 36\}$ for the come ups of the roulette. As known, (see, for example, Pesarin, 1989 or Sheskin, 2000), tests on frequencies are conducted on the assumption that, if the data generating process is with independent and uniformly distributed components, the frequencies counted on a sequence of N elements are uniformly distributed on the k modalities that the phenomenon can take, otherwise some violations will happen with a bigger probability as the discrepancy from the null assumption grows. The observed value for the chi-square test statistic is 35.5956 corresponding to a p -value of 0.4877 for a chi-square distribution with 36 degrees of freedom. Consequently, the afore-said null hypothesis can be accepted with a good confidence. The particular plot of Figure 1, adapted from directional data analysis, allows a visual perception of this unbiasedness in the data generating process and, in particular, that the roulette wheel is not unbalanced.

The succession of Black and Red or Odd and Pair numbers, can be expressed in many ways, one of which is the study of the distribution P of permanencies, i.e. the number of come ups for which a particular colour, or the presence or absence of parity, persists. Consistently with Parzen's (1992) layout, the come up of the number zero interrupts permanencies while its repeatedly coming up does not generate it. The two random variables P_{BR} and P_{OP} which represent, respectively, these permanencies for colours and for parity, are similar and can be identified, both of them, with P (see, among the others, di Bella, 2004):

$$P(P = p) = 2 \left(\frac{18}{37} \right)^{p+1} \frac{19}{37} + \sum_{i=1}^{\infty} \left[\left(\frac{1}{37} \right)^i - \left(\frac{1}{37} \right)^{i-1} \right] I_{(0)}(p)$$

where $I_{(0)}(p) = 1$ if $p = 0$ and $I_{(0)}(p) = 0$ if $p \neq 0$. In order to adequately support some reflections of the subsequent paragraph 3, and as a confirm of the randomness of the come-

Figure 1: *Empirical relative frequencies for the 87090 roulette come ups considered.*



ups, empirical and expected permanencies of Blacks and Reds and of Odds and Pairs are compared. The chi-square test statistic values observed for the two distributions are very low (0.0000606 and 0.0000740 respectively) so that the empirical distribution of permanencies can, in both cases, be considered consistent with the theoretical one.

3. Some gambling systems

In the fascinating history of the Roulette game, many different gambling systems had been developed. Two of the most famous are briefly discussed here: Red and Black (or equivalently Odd and Pair) and the Garçia system (Medail, 1997).

The first one is based on the common conviction that the more Reds (or Blacks) came up, the lower the probability of another Red (o Black) come up is. One way of approaching this system is to bet on the colour opposite to the last which came up. In case of zero, there is no bet until a Red or a Black number comes up. The Garçia system, which assumes that colours tend to come up in succession or in groups of six colours of the same kind, is composed of two parts. At the beginning, a Red and Black system is applied but if three numbers of the same colour came up, then the system supposes that a six colours figure is going to be composed and the betting system acts in that way. In the first part, if the gambler wins, the system stops, while, in the second one, the system stops if the player loses. Consequently, this system can have a variable duration from one (immediate won at the first bet) to six stakes if the player wins the fifth bet. While in the Red/Black system a particular bet amount is not specified, in the Garçia one amounts follow the schema $s, 3s, 7s, 4s, 4s, 3s$, being s a value between the betting minimum and the maximal value which permits to conclude the system with a maximal loss of $15s$.

As well known, if the Roulette wheel is balanced, any system based on delays or permanencies cannot increase the probability of winning and the results of paragraph 2 obviously confirm that. Although the preference of a system to another cannot change the probability of success for a specific bet (for example a single number, a specific colour, etc...), it can

modify the range of wins and losses or the variance of the returns random variable. In di Bella (2004) two gambling systems are compared: the Garçia system and a fixed value system based on the opposite of the colours bet with the Garçia system. The wager of the second system had been decided in order to guarantee the same expected loss of Garçia's one. The player using the Garçia system has big chances of a modest (up to 3s) positive return (0.865) but may have huge losses (up to -15s). The second player has lesser probabilities of win (0.487) but the maximum loss is -2.7s while the maximum win is 10.8s.

4. Conclusions

Although the use of gambling systems cannot invert the disadvantage in winning probabilities terms which the player has in comparison with the bank, the proper allotment of patrimony on numerous bets can modify the mean loss which must be undergone. Every player who wants to execute to infinity a gambling system is designed to ruin but that does not exclude the chance of having profits in a finite time. Most of roulette gambling systems do not analyze the optimal return on the basis of which the player should retire. Gambling systems should be considered mostly as instruments through which differentiate the risk, increasing or decreasing the variance of the patrimonial variance according to the risk attitude which each player has. The analysis of the distribution of real outcomes highlights how the roulette wheel can be effectively considered as an unbiased random numbers generator and the use of gambling systems on those data brought to results consistent with the theoretical ones. This result does not only confirm the genuine randomness of real come ups, but also legitimates the use of computer random numbers generators for any further empirical validation of gambling systems.

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